Sinusoid

A curve similar to the sine function but possibly shifted in phase, period, amplitude, or any combination thereof. The general sinusoid of amplitude a, angular frequency ω (and period $2\pi/\omega$), and phase c is given by

$$f(x) = a\sin(\omega x + c).$$

SEE ALSO: Harmonic Addition Theorem, Simple Harmonic Motion, Sine.
[Pages Linking Here]

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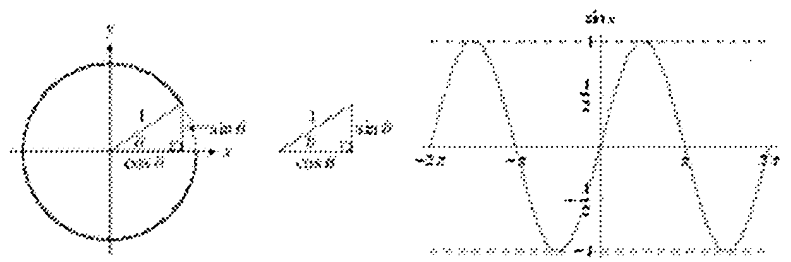
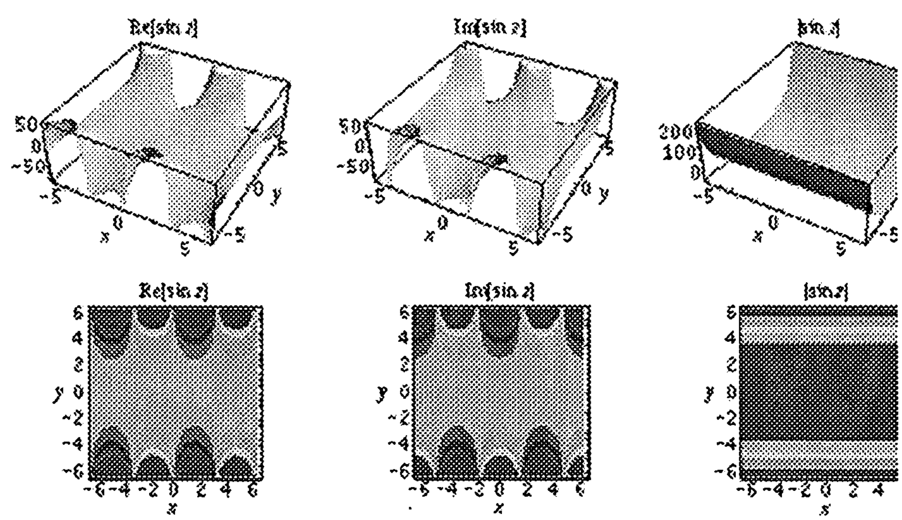
Sine



The sine function $\sin x$ is one of the basic functions encountered in trigonometr others being the cosecant, cosine, cotangent, secant, and tangent). Let θ be an measured counterclockwise from the x-axis along an arc of the unit circle. Ther the vertical coordinate of the arc endpoint. As a result of this definition, the sin function is periodic with period 2π. By the Pythagorean theorem, $\sin\theta$ also obe identity

$$\sin^2\theta + \cos^2\theta = 1.$$



Min Max

Re -2*Pi 2*Pi

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Im -2*Pi 2*Pi Replot

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The definition of the sine function can be extended to complex arguments z, ill
above, using the definition

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i},$$

where *e* is the base of the natural logarithm and *i* is the imaginary number. Sin
entire function and is implemented in *Mathematica* as `Sin[z]`.

A related function known as the hyperbolic sine is similarly defined,

$$\sinh z = \frac{1}{2}(e^{z} - e^{-z}).$$

The sine function can be defined algebraically by the infinite sum

$$\sin x = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)!} x^{2n-1}$$

and infinite product

$$\sin x = x \prod_{n=1}^{\infty} \left(1 - \frac{x^2}{n^2 \pi^2}\right)$$

(Borwein *et al.* 2004, p. 5).

It is also given by the imaginary part of the complex exponential

$$\sin x = \mathrm{I}[e^{ix}].$$

The multiplicative inverse of the sine function is the cosecant, defined as

$$\csc x \equiv \frac{1}{\sin x}.$$

The sine function is also given by the slowly convergent infinite series

$$\sin(z) = -\pi \sum_{k=1}^{\infty} \frac{\mu(k) \ln\left(\frac{n}{k}\right) \operatorname{frac}\left(\frac{kz}{2\pi}\right)}{k \ln n},$$

where $\mu(k)$ is the Möbius function and $\operatorname{frac}(x)$ is the fractional part (M. Trott).

The derivative of $\sin x$ is

$$\frac{d}{dx}\sin x = \cos x,$$

and its indefinite integral is

$$\int \sin x \, dx = -\cos x + C,$$

where C is a constant of integration.

Using the results from the exponential sum formulas

$$\begin{aligned}
\sum_{n=0}^{N} \sin(n x) &= \mathrm{I}\left[\sum_{n=0}^{N} e^{i n x}\right] \\
&= \mathrm{I}\left[\frac{e^{i(N+1)x} - 1}{e^{i x} - 1}\right] \\
&= \mathrm{I}\left[\frac{e^{i(N+1)x/2}}{e^{i x/2}} \frac{e^{i(N+1)x/2} - e^{-i(N+1)x/2}}{e^{i x/2} - e^{-i x/2}}\right] \\
&= \frac{\sin(\frac{1}{2}(N+1)x)}{\sin(\frac{1}{2}x)} \mathrm{I}[e^{i N x/2}] \\
&= \frac{\sin(\frac{1}{2} N x) \sin[\frac{1}{2}(N+1)x]}{\sin(\frac{1}{2}x)}.
\end{aligned}$$

Similarly,

$$\begin{aligned}
\sum_{n=0}^{\infty} p^n \sin(n x) &= \mathrm{I}\left[\sum_{n=0}^{\infty} p^n e^{i n x}\right] \\
&= \mathrm{I}\left[\frac{1 - p e^{-i x}}{1 - 2 p \cos x + p^2}\right] \\
&= \frac{p \sin x}{1 - 2 p \cos x + p^2}.
\end{aligned}$$

The sum of $\sin^2(k x)$ can also be done in closed form,

$$\sum_{k=0}^{N} \sin^2(k x) = \frac{1}{4}\{1 + 2N - \csc x \sin[x(1 + 2N)]\}.$$

The sine function obeys the identity

$$\sin(n\theta) = 2\cos\theta \sin[(n-1)\theta] - \sin[(n-2)\theta]$$

and the multiple-angle formula

$$\sin(n x) = \sum_{k=0}^{n} \binom{n}{k} \cos^k x \sin^{n-k} x \sin[\tfrac{1}{2}(n-k)\pi],$$

where $\binom{n}{k}$ is a binomial coefficient.

A curious identity is given by

$$\frac{\sin(n\,\alpha)}{\sin\alpha} = \sum_{j=1}^{n} \prod_{\substack{k=1 \\ k\neq j}} \frac{\sin(\alpha+\theta_j-\theta_k)}{\sin(\theta_j-\theta_k)}$$

for all α and $\theta_j \neq \theta_k$ (Calogero 1999; Beylkin and Mohlenkamp 2002; Trott 200
6).

Cvijovic and Klinowski (1995) show that the sum

$$S_\nu(\alpha) = \sum_{k=0}^{\infty} \frac{\sin(2\,k+1)\,\alpha}{(2\,k+1)^\nu}$$

has closed form for $\nu = 2\,n+1$,

$$S_{2n+1}(\alpha) = \frac{(-1)^n}{4\,(2\,n)!}\,\pi^{2n+1}\,E_{2n}\left(\frac{\alpha}{\pi}\right),$$

where $E_n(x)$ is an Euler polynomial.

A continued fraction representation of $\sin x$ is

$$\sin x = \cfrac{x}{1+\cfrac{x^2}{(2\cdot 3-x^2)+\cfrac{2\cdot 3\,x^2}{(4\cdot 5-x^2)+\cfrac{4\cdot 5\,x^2}{(6\cdot 7-x^2)+\ldots}}}}$$

(Olds 1963, p. 138). The value of $\sin(2\,\pi/n)$ is irrational for all integers $n > 1$ e
4, and 12, for which $\sin(\pi) = 0$, $\sin(\pi/2) = 1$, and $\sin(\pi/6) = 1/2$, respectively

The Fourier transform of $\sin(2\,\pi\,k_0\,x)$ is given by

$$\begin{aligned}\mathcal{F}_x[\sin(2\,\pi\,k_0\,x)](k) &= \int_{-\infty}^{\infty} e^{-2\pi i k x}\sin(2\,\pi\,k_0\,x)\,d\,x \\ &= \tfrac{1}{2}\,i\,[\delta(k+k_0)-\delta(k-k_0)].\end{aligned}$$

Definite integrals involving $\sin x$ include

$$\begin{aligned}\int_0^\infty \sin(x^2)\,d\,x &= \tfrac{1}{4}\sqrt{2\,\pi} \\ \int_0^\infty \sin(x^3)\,d\,x &= \tfrac{1}{6}\,\Gamma(\tfrac{1}{3}) \\ \int_0^\infty \sin(x^4)\,d\,x &= -\cos(\tfrac{5}{8}\,\pi)\,\Gamma(\tfrac{5}{4}) \\ &= \tfrac{1}{4}\left(\sqrt{5}-1\right)\Gamma(\tfrac{6}{5}),\end{aligned}$$

$$\int_0^\infty \sin(x^5)\, d\, x$$

where $\Gamma(x)$ is the gamma function.

SEE ALSO: Andrew's Sine, Cosecant, Cosine, Elementary Function, Fourier Trans Sine, Hyperbolic Polar Sine, Hyperbolic Sine, Hypersine, Inverse Sine, Polar Sin Function, Sinusoid, Tangent, Trigonometric Functions, Trigonometry.
[Pages Linking Here]

RELATED WOLFRAM SITES:
http://functions.wolfram.com/ElementaryFunctions/Sin/

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